Table 1 Stability criterion test of the distributed parameter model of a slewing flexible beam with noncollocated feedback

Mode number	$\mathrm{Re}\,[(\mathrm{d}p_i/\mathrm{d}k)\vert_{k=0^+}]$
1	−27.405
2	−41.239
3	−35.772
4	−42.973
5	−16.801

Table 2 Closed-loop eigenvalues of the slewing beam in discrete parameter model with three primary modes

Mode	Mode number	Eigenvalue
Primary	1	$-0.0301 \pm 0.301j$
	2	$-0.0233 \pm 10.692j$
	3	$-0.1277 \pm 28.865j$
Residual	4	$-0.0153 \pm 70.155j$
	5	$-1.4552 \pm 327.22j$

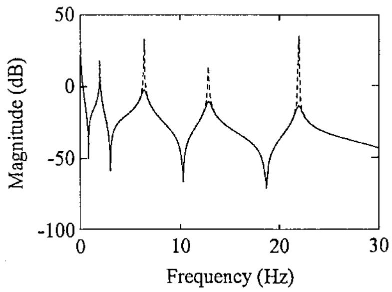

Fig. 1 Bode plot of the slewing flexible beam in distributed parameter model: ——, closed-loop system and - - - -, open-loop system.

to those in Eq. (14), are the hub attitude, angular rate, and tip velocity with the control gains $k_0 = 0.0319$, $k_1 = 0.0266$, and $k_d = 0.016$. Table 2 lists the closed-loop eigenvalues of the discrete model; the loop system is destabilized by the control and observation spillover of residual modes. But this prediction contradicts that in Table 1, which shows that the system is stable. The Bode plot in Fig. 1 shows the distributed parameter model is stable under the same control law and control gains. The example illustrates that inadequate discrete parameter model can lead to incorrect results.

IV. Conclusion

A stability criterion in the frequency domain is developed to predict the closed-loop system stability of a slewing flexible structure in a distributed parameter model. By formulating the open-loop transcendental transfer function, the system dynamics can be fully described so that the spillover problem associated with discrete parameter models can be prevented. The noncollocated feedback controller that uses measurements of the tip displacement and velocity is shown to be effective in vibration suppression; moreover, it is immune from spillover instability. Note that the controller robustness to model uncertainties may require further studies. In the case of discrete system modeling by three primary modes and two residual modes, the closed-loop system under optimal output feedback is predicted to be unstable by eigenvalue calculation. Inadequate discrete parameter model is shown to lead to incorrect results.

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Optimal Control of a Rigid Body with Dissimilar Actuators

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Introduction

SPACECRAFT are frequently controlled by dissimilar actuators such as electrically powered momentum wheels and fuel driven thrusters. In such cases the question arises of how to use the actuators in concert in an optimal manner. Additional difficulties arise when the controls are essentially impulsive, because standard optimization approaches tacitly assume that the controls are bounded or that partial derivatives of the cost with respect to the controls exist.[1] Note that fuel is currently more expensive than electrical power, although this will not be true in every future application, and it will be important to understand the tradeoff.

A geometric approach was used in Refs. 2 and 3 to find the optimal control for a variety of different actuator types. An advantage of this approach is that it easily handles the impulsive solutions associated with minimum fuel controls.[4] This Note applies the geometric approach to a rotating rigid body controlled by dissimilar actuators.

Problem Description

We consider a freely rotating planar rigid body controllable by either or both of two actuators of dissimilar types. The actuators will be referred to as a pair of impulsive thrusters mounted symmetrically about the center of gravity and a torque wheel, although the analysis is applicable to any set of actuators whose cost of operation is proportional to fuel and power, for instance, a cold gas jet and an ion thruster.

The rigid body is rotated about its c.g. through a prescribed angle over a prescribed amount of time. At the end of the maneuver, the body is brought to the origin at rest. The equation governing the planar motion of the rigid body is

$$\theta'' = v_1 + v_2 \tag{1}$$

where θ is the angular position of the rigid body and the nondimensional control inputs are defined by $v_1 = u_1 L T^2/I$ and $v_2 = u_2 T^2/I$. Here the primes denote differentiation with respect to nondimensional time $\tau = t/T$, L is the distance between the thruster and the c.g., u_1 is the force produced by the thrusters, u_2 is the moment produced by the torque wheel, I is the mass moment of inertia of the system, and T is the final time.

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The fuel associated with using the thrusters is proportional to the integral of the absolute value of the control u_1, whereas the cost of using the torque wheel is proportional to the square root of the integral of the control u_2 squared. Thus the total cost associated with the actuators has the general form

$$C = \mu_1 \int_0^1 |v_1|\,\mathrm{d}\tau + \mu_2\left(\int_0^1 v_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}} \tag{2}$$

in which $\mu_1, \mu_2 > 0$ are nondimensional weighting constants. Following the technique of Silverberg and Redmond,[3] our next step is to define the set of allowable controls

$$V = \{v_1, v_2 : C \le 1\} \tag{3}$$

Equation (3) indicates that the cost associated with the admissible controls is bounded above by unity. In customary fashion, Eq. (1) is converted into the state equation

$$x' = Ax + Bv \tag{4}$$

in which $x = [\theta \ \ \theta']^T$, $v = [v_1 \ \ v_2]^T$,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \tag{5}$$

The reachable set associated with Eq. (3) is given by

$$\psi = \int_0^1 e^{-A\tau} Bv\,\mathrm{d}\tau = e^{-A}x_f - x_i = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{6}$$

in which $x_i = [\theta_i \ \ \theta_i']^T$ and $x_f = [\theta_f \ \ \theta_f']^T$ denote the initial and final states, respectively. We assume that the rigid body is finally at rest and located at the origin. Normalizing the initial angular displacement such that $\theta_i = -1$ means that $\psi = [1 \ \ q]^T$ in which $q = \theta_i'/\theta_i$, since the body is finally at rest at the origin.

Form of the Control

We first determine the form of the control that minimizes the cost and transfers the system described by Eq. (1) from its initial state x_i to the origin in unit nondimensional time. Toward this end, we seek[2,3]

$$\bar{\alpha} = \min_{\eta \in H} \max_{v} F = \min_{\eta \in H} \max_{v} \int_0^1 g^T v\,\mathrm{d}\tau \tag{7}$$

where $g(\eta, \tau) = (\eta^T e^{-A\tau} B)^T$. The hyperplane constraint can be expressed as

$$\eta_2 = (1 - \eta_1)/q \tag{8}$$

The form of the control is determined by inspecting F in Eq. (7). Letting $g = [g_1 \ \ g_2]^T$ we find that

$$F = \int_0^1 g^T v\,\mathrm{d}\tau = \int_0^1 g_1 v_1\,\mathrm{d}\tau + \int_0^1 g_2 v_2\,\mathrm{d}\tau$$

$$\le \sup_{0 \le \tau \le 1} |g_1| \int_0^1 |v_1|\,\mathrm{d}\tau + \left(\int_0^1 g_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}} \left(\int_0^1 v_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}} \tag{9}$$

From Eq. (9),

$$F \le \left[\sup_{0 \le \tau \le 1} |g_1| - \frac{\mu_1}{\mu_2}\left(\int_0^1 g_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}}\right] \int_0^1 |v_1|\,\mathrm{d}\tau$$

$$+ \frac{1}{\mu_2}\left(\int_0^1 g_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}} \tag{10}$$

and

$$F \le \frac{1}{\mu_1} \sup_{0 \le \tau \le 1} |g_1|$$

$$+ \left[\left(\int_0^1 g_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}} - \frac{\mu_2}{\mu_1} \sup_{0 \le \tau \le 1} |g_1|\right] \left(\int_0^1 v_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}} \tag{11}$$

Inequalities (10) and (11) can be rewritten in the form

$$F \le [(b_1/\mu_1) - (b_2/\mu_2)]\mu_1 C_1 + (b_2/\mu_2) \tag{12}$$

and

$$F \le (b_1/\mu_1) + [(b_2/\mu_2) - (b_1/\mu_1)]\mu_2 C_2 \tag{13}$$

in which

$$b_1 = \sup_{0 \le \tau \le 1} |g_1|, \qquad b_2 = \left(\int_0^1 g_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}}$$

$$C_1 = \int_0^1 |v_1|\,\mathrm{d}\tau, \qquad C_2 = \left(\int_0^1 v_2^2\,\mathrm{d}\tau\right)^{\frac{1}{2}}$$

Each of the terms in Eqs. (12) and (13) is positive. Thus, if $b_1/\mu_1 < b_2/\mu_2$, then F is maximized when $\mu_1 C_1 = 1$ and $C_2 = 0$. Similarly, if $b_1/\mu_1 < b_2/\mu_2$, then F is maximized when $\mu_2 C_2 = 1$ and $C_1 = 0$. This implies that for any given maneuver and set of weights with $b_1/\mu_1 \ne b_2/\mu_2$, the optimal control will either employ thrusters or torque wheels but not both. In the case where $b_1/\mu_1 = b_2/\mu_2$, it is possible for the optimal control to take a form that employs both actuators, but the cost for such a form will never be less than one of the single control forms.

The optimal control can always employ a single actuator type. Thus, the solution to Eq. (7) can be separated into two possibilities in which either the thrusters alone or the torque wheel alone is employed.

It follows from Eqs. (12), (13), and (7) that

$$\bar{\alpha} = \min_{\eta \in H} \max_{v} F = \min_{\eta \in H} \max(b_1/\mu_1, b_2/\mu_2)$$

$$= \min_{\eta \in H} \max\left[\frac{1}{\mu_1} \sup_{0 \le \tau \le 1} |\eta_2(1 + \tau q) - \tau|,\right.$$

$$\left.\frac{1}{\mu_2}\left(\int_0^1 [\eta_2(1 + \tau q) - \tau]^2\,\mathrm{d}\tau\right)^{\frac{1}{2}}\right] \tag{14}$$

where $g_1 = g_2 = \eta_2(1 + \tau q) - \tau$. Though the details are beyond the scope of this Note, Eq. (14) can be solved analytically for $\bar{\alpha}$. The solution involves evaluation of up to seven candidate solutions, the lowest cost of which is the exact solution.

Results

The solution procedure alluded to in the preceding section was performed letting μ_1, μ_2, and q vary over a large range. Without loss of generality, we restricted our attention to the ratio of costs $\gamma = \mu_1/\mu_2$. These solutions were then compared with explicit expressions developed earlier.[2]

We define $\phi = \tan^{-1}(q)$. Figure 1 summarizes the results for values of ϕ ranging from -90 to 90 deg and for values of γ ranging from 0.8 to 2.2. Since q represents the initial conditions, the vertical axis in Fig. 1 may be thought of as the maneuver to be performed. In particular, $q = 0$ corresponds to rest-to-rest maneuvers, $q > 0$ corresponds to rotation away from the origin, and $q = -1$ corresponds to rotation with a rate that will exactly reach the origin at the final time with no control input.

Several features of this figure are worthy of mention. First note for any cost below $\gamma = 1$ that the thruster is more effective. This corresponds to values of $\mu_1 < \mu_2$ as would be expected from Eq. (2). Similarly, for $\gamma > 2$ the torque wheel is always preferred. This corresponds to $\mu_1 > 2\mu_2$. Also, for rest-to-rest maneuvers, which are those along the line $\phi = 0$, no cases were found where the costs were equal. The switch from thruster to torque wheel occurs when $\gamma = \sqrt{3}$. In the range of $1 < \mu_1/\mu_2 < \sqrt{3}$ it is possible for the optimal control to be either torque wheel or thruster or equal, depending upon the maneuver.

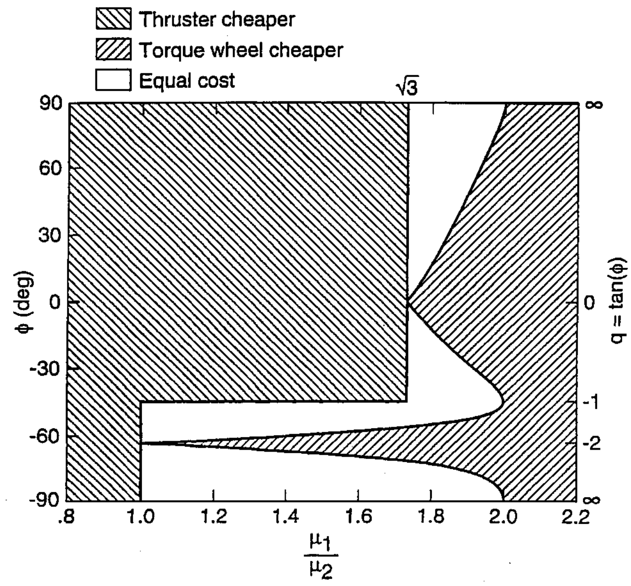

Fig. 1 Optimal control of a rigid body.

Conclusions

It has been shown that for a rigid body with dissimilar actuators that mixed control strategies are nonoptimal. From there it was determined which control is optimal for any given cost weighting and maneuver. This was verified by direct evaluation of the cost of each control. Finally, the optimal control choices for a variety of maneuver/cost weighting pairs were presented. In particular, it was shown when the nondimensional ratio of costs was either more than 2 or less than 1 that the lower weighted control was cheaper regardless of the maneuver. Otherwise, the optimal control choice depended on the maneuver.

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Line-of-Sight Guidance for Descent to a Minor Solar System Body

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Introduction

A PRELIMINARY analysis of line-of-sight guidance for soft landing a payload on a minor solar system body is presented. A simple kinematic model is developed that is used to assess the lander requirements in terms of thrust induced acceleration, thrust vector pointing, and total maneuver Δv. Issues relating to sensor requirements and onboard implementation are also briefly discussed.

Various schemes have been investigated for soft landing payloads on the surface of small asteroids and comets.[1–3] Given the extremely low gravitational acceleration of such bodies, methods have been proposed that are quite different to conventional gravity turn guidance[4] used for powered descent to lunar or planetary surfaces. Surface beacons, descent synchronous with the body rotation, and tethering the vehicle to the asteroid or cometary surface have all been considered.[2,3]

In this Note a scheme is outlined whereby the lander is guided to the desired landing location along a line of sight to the landing location from an orbiter. The orbiter is assumed to have high-resolution imaging capability that may be used to track a desired landing location previously surveyed by the orbiter. The orbiter observes both the lander and the landing location and guides the lander onto the line of sight. A prescribed velocity profile along the line of sight is defined to ensure soft landing. By utilizing available sensors on the orbiter the method allows, in principle, precision soft landing of payloads without the need for complex sensors on the lander. The method may also be used for targeting surface penetrators on hyperbolic fly pasts.

System Kinematics

An orbiter S orbits a small asteroid of radius R at some fixed altitude h above the equator, as shown in Fig. 1. The asteroid is assumed to be spherical and to have a uniform spin rate $\boldsymbol{\Omega}$. After rising above the local horizon of the landing location L, a line of sight is established to an equatorial landing location and the lander P detached from the orbiter. The position vectors of the landing location $\boldsymbol{R}$ and orbiter $\boldsymbol{r}$ may be written in terms of unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ as

$$\boldsymbol{R} = R\cos\lambda\boldsymbol{i} + R\sin\lambda\boldsymbol{j} \tag{1a}$$

$$\boldsymbol{r} = (R+h)\cos(\omega t + \theta_0)\,\boldsymbol{i} + (R+h)\sin(\omega t + \theta_0)\boldsymbol{j} \tag{1b}$$

where the angular velocity of the orbiter relative to the spinning asteroid surface is given by

$$\omega = \sqrt{\frac{GM}{(R+h)^3}} - \Omega, \qquad \omega > 0 \tag{2}$$

and GM is the product of the gravitational constant and the asteroid mass. The line-of-sight vector $\boldsymbol{s}$ from the landing location to the orbiter is then given by

$$\boldsymbol{s} = [(R+h)\cos(\omega t + \theta_0) - R\cos\lambda]\boldsymbol{i} + [(R+h)\sin(\omega t + \theta_0) - R\sin\lambda]\boldsymbol{j} \tag{3}$$

where θ_0 defines the initial orbiter location.

To calculate the required lander acceleration, the line-of-sight angle ϕ must now be obtained. This angle can be obtained from

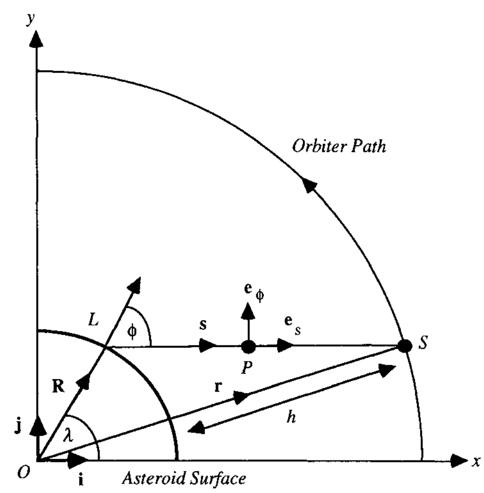

Fig. 1 Schematic geometry of line-of-sight descent.

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